High-Coverage AMS Test Space Optimization by Efficient Parametric Test Condition Generation

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***Abstract*—The characterization of analog-mixed signal (AMS) silicon requires a suitable pattern set able to exercise the parametric operational space to – among other tasks – validate the correct (specified) working behaviour of the device under test. As experience shows, most of the unexpected problems occur for very specific value combinations of a few test condition variables that were not expected to have an influence. Additionally, restrictions on the operational conditions have to be taken into account.**

We present a method to efficiently create a set of test conditions to cover such a constrained search space with a user-defined density. First, an initial test condition set is generated using quasirandom Sobol sequences. Secondly, we analyse the test conditions to identify and fill uncovered areas in the parameter space using the in-house interval constraint solver iSAT3.

The applicability of the method is demonstrated by experimental results on a 19-dimensional search space using a realistic set of constraints.

I. Introduction

Characterization of analog-mixed signal (AMS) circuits is a challenging problem requiring a well-designed set of input patterns [1], [2]. Among other tasks, characterization must validate that the device under test works as specified under all intended operating conditions. Operating conditions are defined by a large number of test condition variables, here called *dimensions*, that can be continuous, discrete or logical. Usually, operating conditions are constrained to not destroy the device (e. g. limit power consumption), avoid undesired behaviour (e. g. frequencies with too many spurs) or avoid unsupported or not implemented modes.

Experience shows that complex chips tend to have unexpected problems that occur for very specific value combinations of a few test condition variables that were not expected to have an influence. The goal of this work is to ensure that such problems will most likely be found, even in a black-box scenario, i. e. a scenario where a priori knowledge is (almost) not available.

As already indicated above experience of the industrial author shows that these unexpected problems depend on only a very few ($n \in \{2, \ldots, 5\}$) test condition variables. Therefore it is not necessary to densely cover the high-dimensional space of all test condition variables, which would be practically impossible, rather it is sufficient to compute a characterization set, that densely covers all n-dimensional subspaces for small n, without leaving "large" untested areas between the (exercised) test conditions. Furthermore, because no a-priori knowledge is available for unexpected problems, all these subspaces should be covered as uniformly as possible. In the context of this work, a good coverage – and thus a good *density* of the characterization set – will be indicated by a small *"Maximum Uncovered n-Cube (MUnC)"* in any n-dimensional subspace.

In the context of AMS circuits, typically constraints defining operational restrictions on the test condition variables exist (e. g. a power limit is provided). In the absence of such constraints, low discrepancy codes, like Sobol sequences [3], provide a fast and effective method to achieve a rather uniform distribution of test conditions in all subspaces [4], [5]. This is no longer guaranteed in the presence of constraints: test conditions violating the constraints have to be excluded because they are outside the given specification. This may cause a distribution bias and an increase in size for the MUnC (illustrated by Example 1 in Sec. II, Fig. 1a). Hence more sophisticated methods are required to be able to take such constraints into account.

This paper presents a method, that:

- generates a set of multi-dimensional test conditions (initial characterization set) using a Sobol sequence and corrects all test conditions violating the user-defined set of linear or non-linear constraints
- finds the maximum uncovered n-cube (MUnC) regarding a given number n of so-called active dimensions to estimate the density of the initial characterization set
- modifies the initial characterization set until a user-defined density is reached by specifically generating test conditions in uncovered regions of the input space.

II. General Problem Description and Approach

We consider AMS circuits having continuous (e. g. waveforms), discrete (e. g. register settings) and logic input parameters as well as static conditions (e. g. device number). Those input parameters – in the following also referred to as *dimensions* – span a multi-dimensional test space.

Example 1. *Let's assume there are 4 dimensions: the voltage (continuous, value range: $[0, 4]V$), the current (continuous, value range: $[0, 5]A$), a mode (discrete; 3 different states) and a Boolean dimension which states whether an embedded component is activated or not.*

The main task of the approach presented in this paper is to get a high quality characterization set for such AMS chips. In the context of black-box characterization it is beneficial to use a characterization set well-distributed over the whole multi-dimensional test condition space in order to uniformly cover all fragments of that space. Thus as a quality measurement the *density* of such a set is used which is defined as the absence of test condition free spaces – so-called *Uncovered n-Cubes (UnC)* – larger than a given size. The size of such an n-cube is the *distance* from its central point to its side. For continuous dimensions the distance is measured in percent of the value range to be able to express this value in multiple dimensions with potentially different value ranges at once. For discrete dimensions it is sufficient to distinguish whether the distance of two values is zero or not. The largest UnC is called *Maximum Uncovered n-Cube (MUnC)*. Consequently a smaller MUnC implies a higher density of the characterization set and vice versa.

A good initial method for characterization set generation is using quasirandom low discrepancy sequences (e. g. Sobol [3], [6], as in this paper). Such sequences are often used for numerical integration, simulation, and optimization [5]. In contrast to pseudorandom numbers, they have the property to efficiently sample even a multi-dimensional space in a more uniform way [4], [7].

However, even this initial characterization set may leave UnCs of certain sizes (e. g. the orange UnC in Figure 1a). Furthermore, usually there are additional dependencies between different input parameters expressed by so-called *parameter constraints*. In theory, those constraints can be arbitrarily complex, including linear and non-linear arithmetic or even transcendental functions. Figure 1a shows such a constraint for Example 1 including initial Sobol generated test conditions. The parameter constraint restricts the power of the chip by demanding $voltage \cdot current < 9W$ and thus excludes the blue area from the test condition space. This invalidates some of the generated test conditions as they violate the parameter constraint like the red point in the figure. Excluding such test conditions affects the density of the characterization set as existing UnCs are enlarged, lowering the quality of the set. An example can be seen in Figure 1a where the existing green UnC grows by the red part because of the removal of the red invalid test condition.

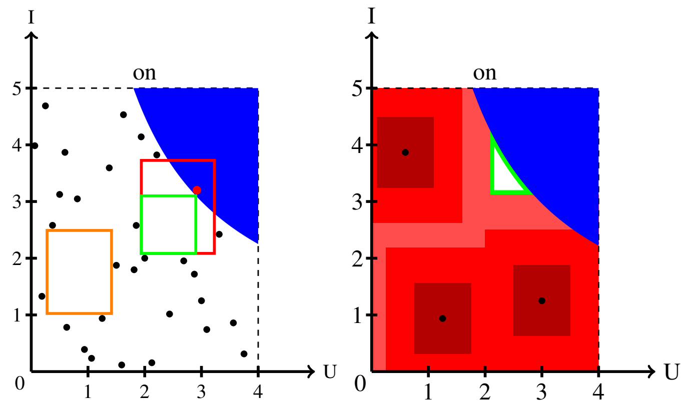
Our approach tackles both issues by filling up the UnCs in the initial characterization set to improve the density and thus the quality.

The general work-flow of the proposed approach is shown in Figure 2. Starting with a set of well-distributed initial test conditions (generated using e. g. quasirandom Sobol sequences or by any other means) the next step is the quality estimation by finding the MUnC considering only a user-given number n of *active* dimensions.

We iteratively generate new test conditions, filling uncovered spaces until a certain density is reached.

Figure 1b shows the general encoding idea for Example 1 with two continuous and one discrete active dimensions including three candidates of the initial characterization set generated by a Sobol sequence.

The red squares around each test condition indicate the area



a) example representation   b) encoding idea, 3rd step ($d = 37.5\%$)

Fig. 1. Example with the parameter constraint $U \cdot I < 9$ and an initial Sobol generated test condition set; active dimensions: U, I, on/off (only "on" is shown)

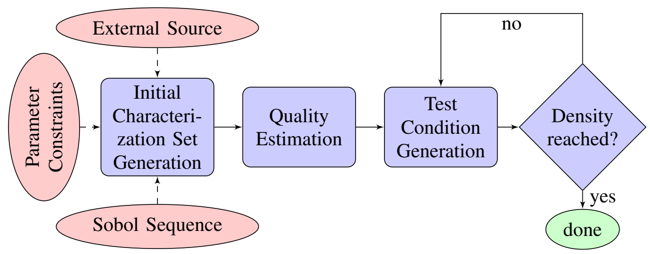


Fig. 2. Work-flow of proposed approach

within the defined distance d. These areas are already covered and considered to be "forbidden". Thus a new test condition is allowed to lie somewhere in the green marked "white space" of the picture. For larger distances or a greater amount of test conditions more and more of the valid space is blocked until at some point it is not possible to find a new test condition with at least the considered distance to all other ones.

This task is encoded into a SAT Modulo Theory (SMT) formula which represents the parameter constraints as well as the "forbidden space" around each existing test condition. The SMT formula is solved by the interval constraint solver iSAT3 [8]–[10].

While a satisfying assignment of the variables provides a new test condition, unsatisfiability of the formula proves its in-existence. In the presented approach the solver can choose any dimension subset of a given size as active dimensions when searching for the location of a new test condition. This ensures that all possible sub-spaces are considered, filled up and hence covered.

The complexity of the proposed problem increases exponentially with the number of dimensions as each new dimension increases the possible test value combinations by at least the factor 2. Considering not all dimensions at the same time reduces the complexity tremendously even though all $\binom{\#dim}{\#actDim}$ possible subsets with a fixed number of dimensions have to be taken into account.

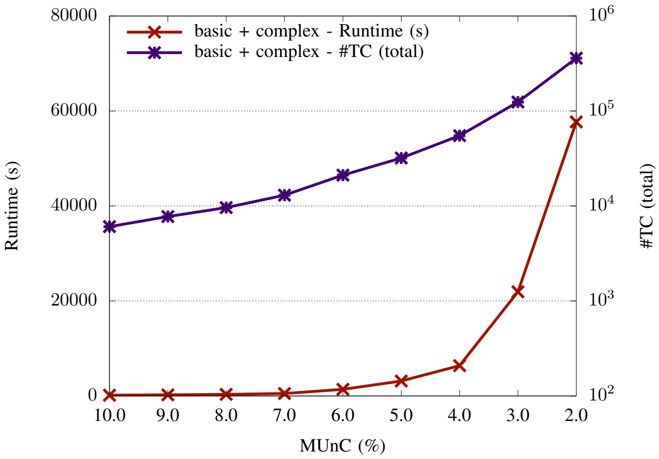


Fig. 3. Runtime (linear scale) and number of generated test conditions (logarithmic scale) for different target MUnC sizes (19 dimensions, 3 active)

III. Experimental Results

The results are based on data from an in-house AMS chip of Advantest. This chip has 9 input parameters containing 4 continuous and 5 discrete dimensions extended by 10 random dimensions (5 continuous, 5 discrete) in order to increase complexity. Furthermore the continuous dimensions are restricted by linear parameter constraints as well as more complex ones including quadratic and even cubic constraints. In general, 3 dimensions are considered to be active. All experiments are performed on an Intel® Xeon® CPU E5-2643 @ 3.30GHz.

Figure 3 shows the runtime and the size of the generated characterization set for multiple experiments with different target densities represented by the MUnC size. The curves illustrate the exponential character of the task in the target density not only for the runtime but also for the number of test conditions. Comparing different numbers of (active) dimensions shows a similar picture. Nevertheless the proposed approach achieves very feasible runtimes, suitable for the application in the industrial context.

As there is – as far as we know – no other approach targeting the same task, the direct competitor is a pure Sobol approach which generates the whole characterization set using a Sobol sequence. As the Sobol approach cannot target for a certain density but only generates a given number of test conditions, the following experimental setup is chosen (see Table I): First the proposed approach is applied for the target MUnC sizes of 10.0% to 2.0% which generates characterization sets of certain magnitudes including $\#TC_{\text{initial}}$ test conditions generated by Sobol. Afterwards a new set with the same size is created by using the pure Sobol approach. The comparison of the MUnC sizes shows the superior quality of the (improved) characterization sets generated by the proposed approach compared to the ones generated using pure Sobol.

IV. Conclusion

We presented a method to generate and improve a parametric characterization set for AMS chips with various kinds of input parameters as well as complex constraints. Starting with a uniformly generated initial characterization set generated using a Sobol sequence, the maximum uncovered n-cube is calculated which is a quality indicator of the characterization set as it approximates the largest uncovered area in the constrained test space. New test conditions are iteratively generated to fill up the characterization set until a user-defined density is reached. The latter two tasks are tackled by an SMT encoding of the problem utilizing the interval constraint solver iSAT3.

The experimental results show the benefits of the proposed method in contrast to a pure Sobol approach: we obtain a far better density for the same set size as well as a method for measuring and selectively improving the quality of the characterization set and all of that in a feasible amount of time.

	proposed approach			pure Sobol approach	
	MUnC (%)	$\#TC_{\text{initial}}$	$\#TC_{\text{total}}$	#TC	MUnC (%)
19 dimensions	10.0	1500	6560	6560	22.0
	9.0	2057	7716	7716	21.0
	8.0	2929	10219	10219	20.5
	7.0	4373	16211	16211	18.5
	6.0	6944	21554	21554	17.0
	5.0	12000	33632	33632	14.0
	4.0	23437	59041	59041	13.5
	3.0	55555	132382	132382	12.5
	2.0	187500	375911	375911	7.5

TABLE I
Density comparison between Sobol and the proposed approach for the same amount of test conditions

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